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Certified Actuarial Analyst Resource Guide

Module 4

2017

caa-global.org

Contents

Welcome to Module 4 - Short Term Actuarial Mathematics of the Certified Actuarial Analyst qualification

The Certified Actuarial Analyst (CAA) is a professional qualification from CAA Global.

It is designed to provide you with a technical skills qualification if you:

- work alongside actuaries – in areas such as data analysis, pricing and marketing
- work in the wider financial services area – perhaps you already have other qualifications, and would like to develop a skill set that will mark you out in a competitive environment
- work in a service centre environment – the analytical skills you'll learn can then be added to your business knowledge
- have strong maths skills, and you want to learn on the job rather than going to university.

The aim of the Module 4 Short Term Actuarial Mathematics exam is to provide a further grounding in mathematical and statistical techniques of particular relevance to non-life insurance.

This Resource Guide for Module 4 gives you the syllabus you will cover for the exam, and details of some online and other resources that will help you study for the Module 4 exam. There is also a specimen exam paper giving examples of the type of questions you will be asked.

Additional information about the Module 4 exam, including:

- How to enter for the exam
- What will happen at the exam centre

can be found in the:

- Student Actuarial Analyst Handbook

If you have any further questions contact the CAA Administration Team who will be happy to help

Email the team at:

enquiries@caa-global.org

The Certified Actuarial Analyst qualification

There are seven exams which you will need to complete for the qualification:

Module title	Assessed by
Fundamental Mathematics & Statistics – Module 0	2 hour Computer Based Assessment
Finance and Financial Mathematics – Module 1	2 hour Computer Based Assessment
Statistics and Models – Module 2	2 hour Computer Based Assessment
Long Term Actuarial Mathematics – Module 3	2 hour Computer Based Assessment
Short Term Actuarial Mathematics – Module 4	**2 hour Computer Based Assessment of 60 questions**
Models and Audit Trails – Module 5	3 hour examination
Online Professional Awareness Test (PAT)	90 minute examination

In addition to passing the above exams, you must complete at least one year of relevant work experience (Work-based skills).

The syllabus for the Module 4 exam

The mathematical topics covered by the Module 4 exam are:

- Probabilities and moments of loss distribution both with and without limits and risk-sharing arrangements
- Risk models involving frequency and severity distributions
- Ruin
- Techniques for analysing a run-off (or delay) triangle and projecting the ultimate position

You can find the full Module 4 syllabus in Appendix 1 of this Resource guide.

Assessment of the Module 4 exam

The Module 4 exam is assessed by a 2 hour computer based exam containing 60 multiple choice questions.

Before you start your 2 hour exam you will have an extra 15 minutes for:

- reading the exam instructions, and
- working through some basic sample questions so that you become familiar with the format of the exam.

You will also need to sign a statement of confidentiality in relation to the exam materials.

Pass standards for the exam are set by CAA Global. Details of pass standards for CAA exams will be published in due course.

Pearson | VUE

You can sit the exam at one of the many centres worldwide managed by Pearson VUE.

You can find details of your local exam centre on their website by using their regional contact details:

www.pearsonvue.com/caaglobal

Studying for the Module 4 exam

Recommended study hours

We recommend that you spend 125-150 hours studying to prepare for the exam.

Tuition

BPP ACTUARIAL EDUCATION

BPP Actuarial Education (ActEd), provides online study material for this exam.

Details of their training materials and services are available on their website.

Website: **www.bppacted.com**

Email: **ActEd@bpp.com**

Tel: +44 (0)12346 550 005

Specimen exam paper

A specimen exam paper for Module 4, with sample exam questions, is given in Appendix 2. This will show you the types of questions which will be asked in the exam.

Please note

Only 20 sample questions are given in the Specimen exam paper. There will be a total of 60 questions in the Module 4 exam you sit.

The questions found in the specimen paper will not be included in the Module 4 exam.

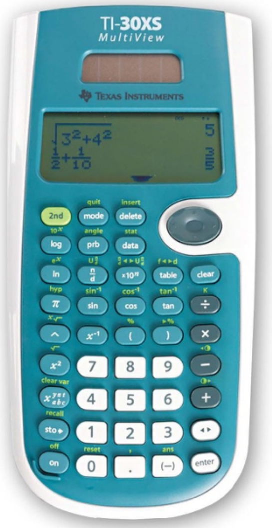
Resources available at the exam centre

Calculators

There is only one authorised calculator for all the CAA exams:

- **Texas Instruments TI-30 Multiview** (with or without suffix).

You should bring your own calculator with you to the exam. It will be checked by exam centre staff, and the memory will be cleared.



If you bring a different calculator model it must be left in the locker with your other personal belongings.

An on-screen scientific calculator will also be available for you to use during the exam. However, some students have reported that they found this on-screen calculator difficult or cumbersome to use, and so you may prefer to take your own TI-30 calculator to the exam with you.

The TI-30 Multiview calculator is available to buy from shops or online retailers.

To clear the memory and reset the calculator:

[2nd] [reset] 2 or [on] & [clear]	Resets the TI-30XS MultiViewTM Calculator. Returns unit to default settings; clears memory variables, pending operations, all entries in history, and statistical data; clears the constant feature, K and Ans

Making notes during the exam

You will be provided with an erasable note board at the centre to use during the exam.

You will only be given one board at a time but are entitled to as many as you need during the exam, and you will be able to keep these at your desk for the duration of the exam. You should ask the supervisor for more if needed. The Pearson VUE staff will not provide you with an eraser for the note boards.

The note boards will be collected by Pearson VUE staff at the end of the exam.

On occasion you may instead be given scrap paper to make notes on.

Formulae and Tables for actuarial examinations

The book of Formulae and Tables for examinations has been published to help students who sit actuarial exams.

The book gives you formulae for:

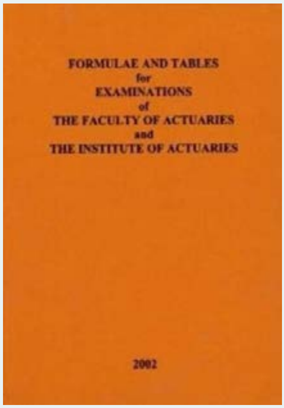
- selected mathematical and statistical methods,
- calculus, time series and economic models, and many other topics.

There are also tables for:

- compound interest calculations,
- selected statistical distributions, and
- other actuarial calculations.

You should make yourself familiar with these tables and formulae during your exam preparation.

You will not be able to use your own copy of the book during your exam, but a PDF copy will be available on your exam screen for you to use.



You can buy a copy by following this link: **www.actuaries.org.uk/catalog/formulae-and-tables**

Appendix 1
Syllabus: Module 4 - Short Term Actuarial Mathematics

Aim

The aim of the Short Term Actuarial Mathematics syllabus is to provide a further grounding in mathematical and statistical techniques of particular relevance to non-life insurance.

Topic 1

Probabilities and Moments of loss distribution both with and without limits and risk-sharing arrangements

Indicative study and assessment weighting 25%

1

Learning objectives

(i) Describe the properties of the statistical distributions which are suitable for modelling individual and aggregate losses.

(ii) Estimate the parameters of a failure time or loss distribution when the data is complete, or when it is incomplete, using maximum likelihood and the method of moments.

(iii) Apply the principles of statistical inference to select suitable loss distributions for sets of claims.

(iv) Define moments and moment generating functions (where defined) of loss distributions including the gamma, exponential, Pareto, generalised Pareto, normal, lognormal and Weibull distributions.

(v) Explain the concepts of excesses, deductibles, and retention limits.

(vi) Describe the operation of simple forms of proportional and excess of loss reinsurance.

(vii) Calculate the moments of the claim amounts paid by the insurer and the reinsurer in the presence of excesses (deductibles) and reinsurance.

Topic 2

Risk models involving frequency and severity distributions

Indicative study and assessment weighting 25%

2

Learning objectives

(i) Determine models appropriate for short term insurance contracts in terms of the numbers of claims and the amounts of individual claims.

(ii) Describe the major simplifying assumptions underlying the models in 2(i).

(iii) Derive the moment generating function of the sum of N independent random variables; in particular when N has a binomial, Poisson, geometric or negative binomial distribution.

(iv) Define a compound Poisson distribution and apply the fact that the sum of independent random variables each having a compound Poisson distribution also has a compound Poisson distribution.

(v) Calculate the mean and variance for compound binomial, compound Poisson and compound negative binomial random variables and derive the coefficient of skewness for the compound Poisson case and make a comment on the sign of the skewness in the other case.

(vi) Derive formulae for the moment generating functions and moments of aggregate claims over a given time period for the models in 2(i) in terms of the corresponding functions for the distributions of claim numbers and claim amounts, stating the mathematical assumptions underlying these formulae.

(vii) Calculate the mean and variance for compound binomial, compound Poisson and compound negative binomial random variables for both the insurer and the reinsurer after the operation of simple forms of proportional and excess of loss reinsurance and derive the coefficient of skewness for the compound Poisson case and make a comment on the sign of the skewness in the other case.

Topic 3
Ruin
Indicative study and assessment weighting 25%

3

Learning objectives

(i) Explain what is meant by the aggregate claim process and the surplus process for a risk.

(ii) Calculate probabilities of the number of events in a given time Interval and waiting times using the Poisson process and the distribution of inter-event times to probabilities.

(iii) Calculate, using the Poisson process and distribution of inter-event times, probabilities involving waiting times and the number of events in a given time interval.

(iv) Define the adjustment coefficient for a compound Poisson process.

(v) Calculate the adjustment coefficient for a compound Poisson process in simple cases.

(vi) Define the probability of ruin in infinite/finite and continuous/discrete time.

(vii) Explain relationships between the different probabilities of ruin.

(viii) State Lundberg's inequality.

(ix) Explain the significance of the adjustment coefficient in Lundberg's inequality.

(x) Describe the effect on the probability of ruin, in both finite and infinite time, of changing parameter values.

(xi) Determine the effect on the adjustment coefficient and hence on the probability of ruin of simple reinsurance arrangements

Topic 4
Techniques for analysing a run-off (or delay) triangle and projecting the ultimate position
Indicative study and assessment weighting 25%

4

Learning objectives

(i) Describe how a statistical model can be used to underpin a run-off triangles approach.

(ii) Define a development factor.

(iii) Demonstrate how a set of assumed development factors can be used to project the future development of a run-off triangle

(iv) Apply the basic chain ladder method for completing the run-off triangle.

(v) Demonstrate how the basic chain ladder method can be adjusted to make explicit allowance for inflation.

(vi) State alternative ways for deriving development factors which may be appropriate for completing the run-off triangle.

(vii) Apply the average cost per claim method for estimating outstanding claim amounts.

(viii) Apply the Bornhuetter-Ferguson method for estimating outstanding claim amounts.

(ix) State the assumptions underlying the chain ladder, average cost per claim and Bornhuetter-Ferguson methods.

END OF SYLLABUS

Appendix 2
Specimen Examination Paper: Module 4 - Short Term Actuarial Mathematics

Please note:

- This Specimen Examination Paper acts as an example of the types of questions that will appear in the Module 4 Exam Paper.
- Although only 20 sample questions are listed in this specimen paper, there will be a total of 60 questions in the actual Module 4 examination.
- The questions found in the Module 4 Specimen Examination Paper will not be included in the actual Module 4 examination.

1. The following sample of claim amounts is obtained from a Pareto claims distribution with parameters α and λ:

15, 20, 28, 30, 35, 60, 80, 200, 220, 400

If it is given that $\sum x^2 = 261{,}934$, use the method of moments to **estimate** the values of α and λ.

A. $\alpha = 3.65$ and $\lambda = 288.32$

B. $\alpha = 5.33$ and $\lambda = 471.10$

C. $\alpha = 11.52$ and $\lambda = 1144.57$

D. $\alpha = 108.8$ and $\lambda = 26{,}193.4$

Answer: **C**

2. Which of the following is **not an assumption used to simplify** the short-term insurance model?

A. Moments and distribution of claim costs is known with certainty.

B. Moments and distribution of claim numbers is not known with certainty.

C. Absence of allowance for administration and underwriting expenses.

D. There are no delays in the settlement of claims.

Answer: **B**

3. An insurance company writes three types of policy: car, bike and fire. For each type, the number of claims the company expects to receive in any given year is modelled by a Poisson distribution. These distributions have different means, as displayed in the table below.

Portfolio	Mean
Car insurances	10
Bike insurances	12
Fire insurances	21

Denote by N the total number of claims these three portfolios will generate together during a period of two years. What distribution does N have?

A. $N \sim \text{Poisson}(43)$
B. $N \sim \text{Poisson}(86)$
C. $N \sim \text{binomial}(43, 97/420)$
D. $N \sim \text{binomial}(86, 97/420)$

Answer: **B**

4. The number of claims an insurance company receives per year is denoted by the random variable N.

The size of each individual claim is denoted by the random variable X.

An actuary uses the formula $E(S) = E(N).E(X)$ to calculate the first moment of the distribution of S, the total claims an insurance company receives per year.

Identify which of the following assumptions the actuary has made in applying this approach.

A. The distribution of the number of claims N is Poisson.

B. The number of claims N and the size of each individual claim X are independent.

C. The size of each claim X is a discrete random variable.

D. The size of each claim X is a continuous random variable.

Answer: **B**

5. Denote:

- R is the adjustment coefficient of a compound Poisson process
- λ is the mean number of claims for the Poisson process
- $M_X(t)$ is the moment generating function of the random variable X of claim amounts
- c is the rate of premium income per unit time $= (1+\theta)\lambda E(X)$
- $\theta\ (>0)$ is the premium loading factor

Which of the following statements is **not** true?

A. $R < 2\theta\, E(X) / E(X^2)$

B. R depends on the value of λ.

C. R is a measure of risk associated with the process.

D. R is the unique positive root of the equation: $\lambda M_X(R) = \lambda + cR$

Answer: **B**

6. The number of claims received in an insurance company over a period *t* has a Poisson distribution with parameter 10. The claims amounts have a gamma distribution with parameters (1, 0.005). Assume independence between individual claims.

Calculate the adjustment coefficient using a premium loading factor of 50%.

A. -0.095

B. -0.06167

C. 0.001667

D. 0.01667

Answer: **C**

7. The aggregate claims for a portfolio of insurance policies follow a compound Poisson process with parameter $\lambda = 20$. The individual claim amounts are £100 with probability 0.25 and £200 with probability 0.75. The initial surplus is £1,000.

Assume a normal approximation for total claims.

What is the premium loading factor such that the probability of ruin by time 3 is 0.05?

A. 0.1233
B. 0.1234
C. 0.1235
D. 0.1240

Answer: **C**

8. **Identify** which of the following statements is Lundberg's inequality.

A. $\Psi(U) < \exp(-RU)$

B. $\Psi(U) > \exp(-RU)$

C. $\Psi(U) \leq \exp(-RU)$

D. $\Psi(U) \geq \exp(-RU)$

Answer: **C**

9. Which of the following statements best explains why run-off triangles typically arise in non-life insurance?

A. A claim occurring does not bring the policy to an end.
B. Claims may occur at any time during the policy period.
C. Cover is normally for a fixed period, most commonly one year, after which it has to be renegotiated.
D. It may take some time after a loss until the full extent of the claims which have to be paid are known.

Answer: **D**

10. **Consider** the following cumulative claims payments, and **calculate** the reserve to be held against the 2014 accident year, using the smallest development ratio as the development factor for each development year:

AY/DY	0	1	2	3	Ult
2011	500	700	950	1020	**1020**
2012	300	500	710		
2013	450	560			
2014	600				

A. 482.6
B. 572.5
C. 836.8
D. 1082.6

Answer: **A**

11. The pooled one-year development factors from development year 0 to 1, 1 to 2 and 2 to 3 respectively are 1.02, 1.04, and 1.03.

The historical ultimate loss ratio for all origin years of the claims triangle in question is 0.98.

Calculate the two-year development factor from development year 1 to 3 using the basic chain ladder method.

A. 1.0498
B. 1.0708
C. 1.0712
D. 1.0926

Answer: **C**

12. The tables below show the cumulative number of claims and the average cost per claim in accident years 2017 and 2018.

Cumulative Number of Claims		Development Year 0	1
Accident Year	2017	200	220
	2018	250	

Cost per Claim		Development Year 0	1
Accident Year	2017	5.2	5.5
	2018	6.5	

What is the ultimate cumulative expected claim amount in respect of the 2018 accident year using the average cost per claim method?

A. 1625
B. 1719
C. 1891
D. 3101

Answer: **C**

13. Which of the following is **true** about the coefficient of skewness formula and category of skewness for a compound negative binomial distribution?

	Formula	Category
A.	$E[(S-E(S))^3] / [var(S)]^{3/2}$	positively skewed
B.	$E[(S-E(S))^3] / [var(S)]^{3/2}$	negatively skewed
C.	$E[S-E(S)] / [var(S)]^{3/2}$	negatively skewed
D.	$E[(S-E(S)]^3) / [var(S)]^3$	positively skewed

Answer: **A**

14. **Identify** the correct expression for the moment generating function (MGF) for aggregate claims S where X is the individual claim amount and N is the number of claims (where N follows a binomial distribution with parameters n and p). The MGF for X may be taken as $M_X(t)$.

A. $M_S(t)=(pM_X(t)+(1-p))^n$
B. $M_S(t)=(M_X(t)+(1-p))^n$
C. $M_S(t)=(M_X(t)+(1-p))$
D. $M_S(t)=((pM_X(t)+(1-p))^{n-1}$

Answer: **A**

15. Under a particular individual excess of loss arrangement, the reinsurance layer is set at £25,000 in excess of £30,000.

If there are three claims worth £10,000, £47,000 and £65,000 in one particular day, how much will the reinsurer pay in total in respect of these claims?

Ignore any excesses in respect of the policyholder.

A. £42,000
B. £52,000
C. £62,000
D. £80,000

Answer: **A**

16. Let $X \sim Exp(\lambda)$ and $Y \sim Gamma(2, \lambda)$. Let $M_X(t)$ and $M_Y(t)$ denote the moment generating function of X and Y respectively.

Which of the following statements is **true**?

A. $E[Y] = (E[X])^2$
B. $V[Y] = [V(X)]^2$
C. $M_X(t) = 0.5\, M_Y(t)$
D. $M_X(t) = \sqrt{M_Y(t)}$

Answer: **D**

17. A motor insurance company models the cost of each property damage claim, X, using an exponential distribution with rate parameter λ.

Express the probability of a property damage claim being greater than £200 and less than £600 in terms of:

i) its distribution function, F(x) and
ii) an explicit formula.

A. i) F(200) + F(600) ii) $e^{-600\lambda} - e^{-200\lambda}$
B. i) F(200) - F(600) ii) $e^{-200\lambda} - e^{-600\lambda}$
C. i) F(200) - F(600) ii) $e^{-600\lambda} - e^{-200\lambda}$
D. i) F(600) - F(200) ii) $e^{-200\lambda} - e^{-600\lambda}$

Answer: **D**

18. 100 samples are obtained from an exponential distribution with unknown rate parameter λ. The sum of all samples equals 158.3.

Calculate the maximum likelihood estimate for λ based on these samples.

A. 0.006
B. 0.632
C. 1.583
D. 158.3

Answer: **B**

19. **Consider** the following incurred claims triangle for a commercial property line of business (figures in £s):

	Development Year	1	2	3	4	5	6
Accident Year	2013	300	470	520	550	560	565
	2014	500	900	1200	1300	1350	
	2015	600	800	900	950		
	2016	800	1600	2500			
	2017	1000	3550				
	2018	1500					

and the respective earned premiums and the paid claims to date (figures in £s):

Accident Year	Earned Premium	Paid Claims to Date
2009	1000	500
2010	2000	900
2011	2500	700
2012	3000	1500
2013	3500	2000
2014	4000	200

Market benchmarks indicate that the commercial property line of business ultimate loss ratio is approximately 75%.

Estimate the total outstanding claims amount using the Bornhuetter-Ferguson method.

A. £13,277

B. £7,953

C. £7,212

D. £1,585

Answer: **B**

20. A homogeneous Poisson process has events occurring at a rate of 12 per hour.

Calculate the probability that the waiting times between events 2 and 3, and between events 3 and 4, are both greater than 15 minutes.

A. 0.0025

B. 0.0498

C. 0.9029

D. 0.9975

Answer: **A**

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